Solutions

2.1: Instantaneous Rate of Change

Definition: The **instantaneous rate of change** of f at a, is defined to be the limit of the average rates of change of f over shorter and shorter time intervals around a. We can write this mathematically as

$$\underset{\text{“}b\text{ approaches }a\text{”}}{\lim_{b\to a}} \frac{f(b)-f(a)}{b-a}.$$

Definition: The **derivative** of f at a, written $f'(a)$, is defined to be the instantaneous rate of change of f at the point a. It is common to write the derivative mathematically as

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}.$$

Question: Explain in your own words how the two limits above actually represent the same thing, i.e., explain how they are equal.

Remark: It is common that we will see

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

but this should not confuse you at this point since the variable x (and h) above is (are) just “dummy variables.”

Exercise 1: In a time of t seconds, a particle moves a distance of s meters from its starting point, where $s=4t^2+3$.

(a) Find the average velocity between $t=1$ and $t=1+h$ if:

$$(i)\quad h=0.1,\qquad (ii)\quad h=0.01,\qquad (iii)\quad h=0.001.$$

(b) Use your answers to part (a) to estimate the instantaneous velocity of the particle at time $t=1$.

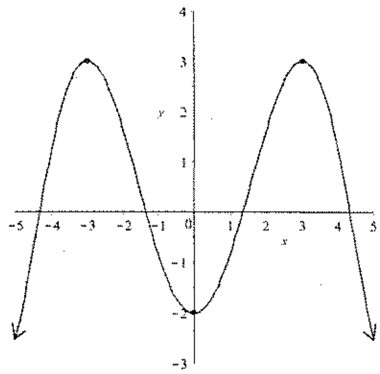
(i) $\dfrac{4(1.1)^2+3-(4(1)^2+3)}{.1}=8.4$ (ii) $\dfrac{4(1.01)^2+3-(4(1)^2+3)}{.01}=8.04$

(iii) $\dfrac{4(1.001)^2+3-(4(1)^2+3)}{.001}=8.004$

(b) Outputs approach 8 as h approaches 0

so inst velocity = 8

Exercise 2: Consider the function f given by the graph below. For each pair of numbers, determine which is larger.



(a) $f(3)$ or $f(4)$

(b) $f(4) - f(3)$ or $f(4) - f(2)$

(c) $\frac{f(4)-f(3)}{4-3}$ or $\frac{f(4)-f(2)}{4-2}$

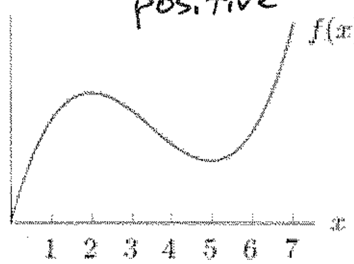
(d) $f'(3)$ or $f'(4)$

Exercise 3: The graph of a function $y = f(x)$ is shown below. Indicate whether each of the following quantities is positive or negative and illustrate your answers graphically.

(a) $f'(1)$ positive

(b) $\dfrac{f(3) - f(1)}{3 - 1}$ positive

(c) $f(4) - f(2)$ negative

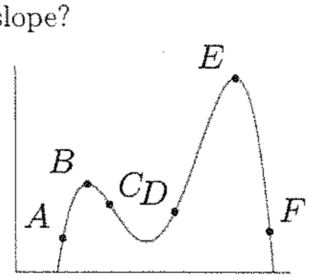


Exercise 4: For the function below, at what labeled points is the slope of the graph positive? Negative? At which labeled point does the graph have the greatest slope? The least slope?

Positive:
A, D

Negative:
C, F

Zero:



Largest = A
Smallest = F